A statement is a group of words and symbols that can be classified as true or false.

Chicago is located in the state of Illinois.
  True statement

$X < 6$ when $x = 10$
  False statement

Get out of here!
  Not a statement (is neither true nor false)

Open statement or open sentence:

Example: $X + 3 = 10$
  Can be either true or false, depending on the replacement value for the variable $x$. If $x = 7$, the statement is true; if $x = 5$, the statement is false.

Notice: $2x = x + x$
  Not open since true for any value for $x$. 
We can use letters to represent statements:
- GW Bush is the President of the US.  \( P \)
- All triangles have four sides. \( Q \)

We can create a new statement:
GW Bush is the President of the US and all triangles have four sides.
\( P \) and \( Q \)

Statement is false. Why?

In form: \( P \) and \( Q \). Both \( P \) and \( Q \) must be true for the conjunction to be true.

Conjunction:

Disjunction:
In form: \( P \) or \( Q \). Only false when both \( P \) and \( Q \) are false.
- All dogs are mammals. \( P \)
- It is raining outside. \( Q \)

The disjunction: \( P \) or \( Q \) states:
All dogs are mammals or it is raining outside.
The disjunction is true. Why? Does the actual weather today matter?

Negation of a statement:
makes a claim opposite of the original statement
- My snake’s name is George. \( P \)
- My snake’s name is not George. \( \neg P \) read “not P”
- All spaghetti is white. \( Q \)
- Some spaghetti is not white. \( \neg Q \) read “not Q”
Conditional Statement: (or Implication) \[\text{If } P \text{ then } Q\]

- If Betty eats a strawberry she will get hives.
- If an animal can swim, it is a fish.
- If the phone rings, you have a call.
- If a figure is a square, it has four sides.

Statement \(P\) is called the hypothesis. Statement \(Q\) is called the conclusion.

Reasoning

- **Intuition**: An inspiration leading to the statement of a theory.
- **Induction**: An organized effort to test the theory. (Observation & collecting data)
- **Deduction**: A formal argument that proves the tested theory. The knowledge and acceptance of selected assumptions guarantees the truth of the conclusion.

Valid Argument

An argument in which the conclusion follows logically from previously stated and accepted premises or assumptions.

- If the last digit of a number is 2, then the number is even.
- The last digit of the number is 2.

Therefore, the number is even.
Law of Detachment

Let P and Q represent simple statements, and assume that statements 1 and 2 are true. Then a valid argument having conclusion C has the form:

1. If P, then Q premise
2. P premise
C. ∴ Q conclusion

Note: The symbol ∴ means “therefore”

How does the previous example fit the form of the Law of Detachment?

1. If the last digit of a number is 2, then the number is even.
2. The last digit of the number is 2.
C. ∴ The number is even.

What are the simple statements P and Q?
What is the conditional statement If P then Q?

Is the following argument valid? Assume that premises 1 and 2 are true.

1. If it is raining, then Tim will stay in the house.
2. It is raining
C. ∴ Tim will stay in the house.

What are the simple statements P and Q?
What is the conditional statement If P then Q?
Invalid argument

How does this example not fit the format of the Law of Detachment?

1. If a man eats hotdogs he is not a vegetarian.
2. The man is not a vegetarian.
C. ∴ The man eats hotdogs.

Counterexample?

Invalid Argument

1. If P, then Q
2. Q
C. ∴ P

This is the common error of asserting the conclusion.

Geometry Example

1. If an angle is a right angle, then it measures 90°
2. Angle A is a right angle.
C. Angle A measures 90°

What are simple statements P and Q? What is the conditional statement if P then Q? Is this argument valid?